Interpolation goes on SAFARI

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University of Lugano, Switzerland

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Context: Software model checking

Software model checking:

- Given a program P and a (safety) property ϕ, does P exhibit an execution violating ϕ?

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- Transition-relation representation of input program
- Lazy Abstraction [HJMS02]
  - different degrees of precision for different parts of the program
- Interpolation-based refinement [HJMM04, McM06]

Lazy Abstraction with Interpolants

[McM06]

Original (Concrete) Program

```
1:   y = x;
2:   while ( x ≥ 1 ) {
3:         x = x - 1;
4:         y = y - 1;
5:   }
6:   if ( y ≥ 1 )
7:         ERROR;
```

Lazy Abstraction with Interpolants

[McM06]

Original (Concrete) Program

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1:  y = x;
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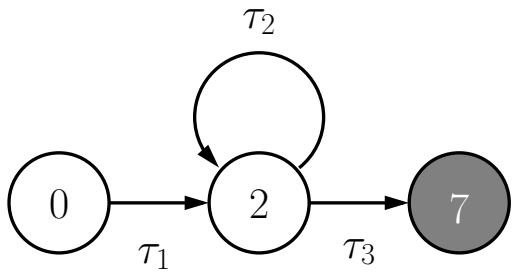
Transition System (over $\mathcal{LIA}$)



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$$\tau_3\colon x \leq 0 \wedge y \geq 1 \wedge \left\{ \begin{array}{l} x' := x \\ y' := y \end{array} \right.$$

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(Abstract) Program Unwinding

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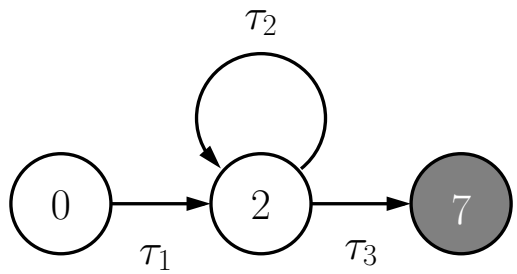

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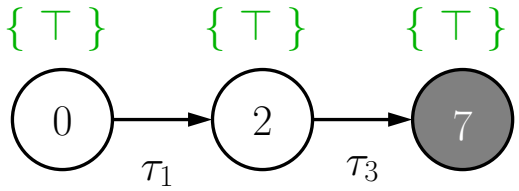

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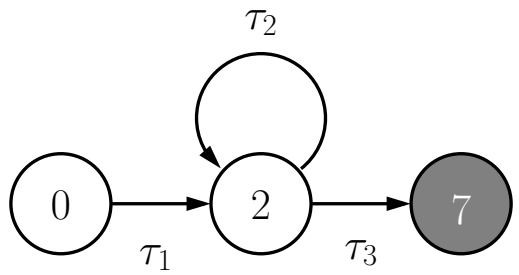

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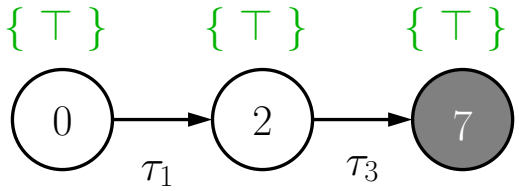

true

$x_1 = x_0$

$y_1 = x_0$

$x_1 \leq 0$

$y_1 \geq 1$

$x_2 = x_1$

$y_2 = y_1$

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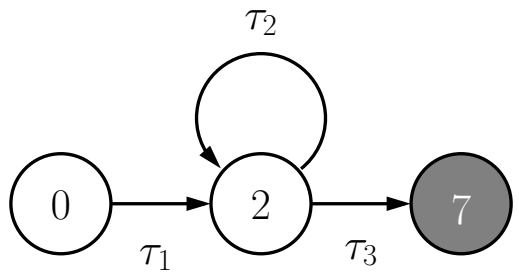

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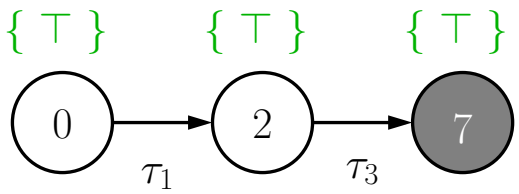

$\{ \top \}$

true

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$y_1 = x_0$

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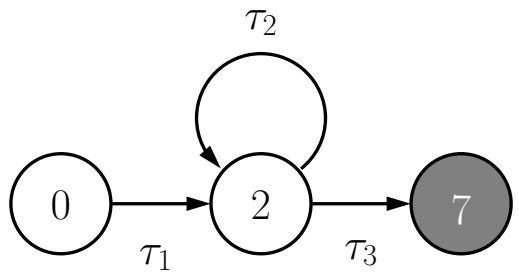

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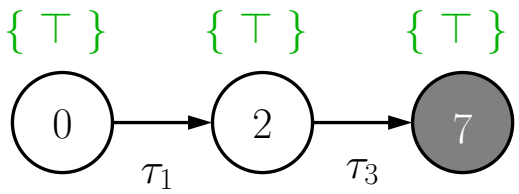

$\{ \top \}$

true

$x_1 = x_0$

$y_1 = x_0$

$\{ y_1 - x_1 \leq 0 \}$

$x_1 \leq 0$

$y_1 \geq 1$

$x_2 = x_1$

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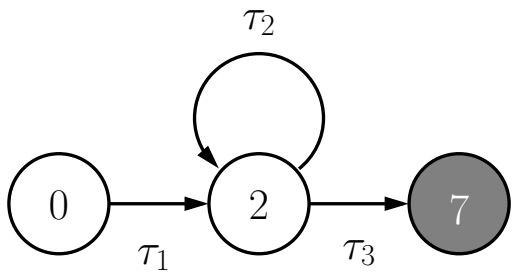

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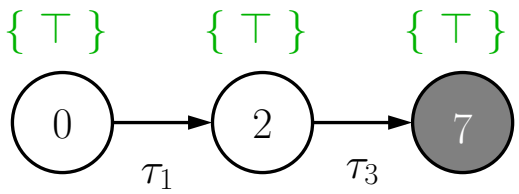

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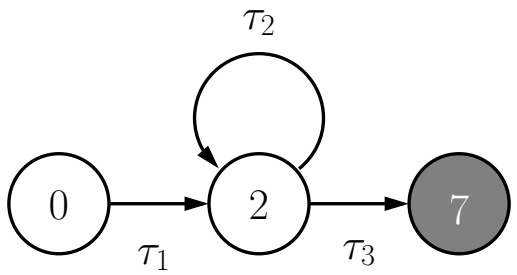

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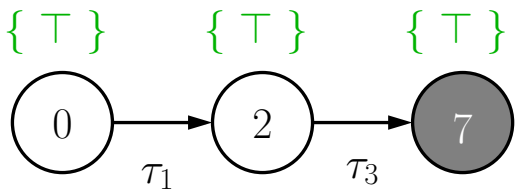


$A \Rightarrow I$

$I \wedge B$ is unsat

common vocabulary

{ ⊤ }

true

$x_1 = x_0$

$y_1 = x_0$

{ y_1 - $x_1 \leq 0$ }

$x_1 \leq 0$

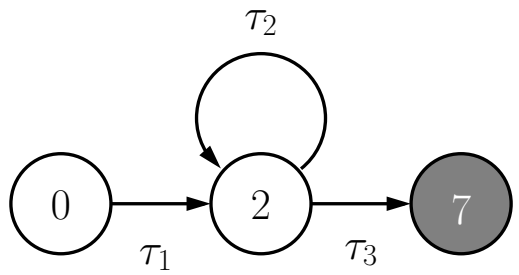
$y_1 \geq 1$

$x_2 = x_1$

$y_2 = y_1$

{ ⊥ }

Transition System (over $\mathcal{LIA}$)



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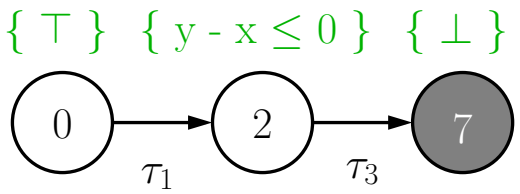

$\{ \top \}$
$\quad \text{true}$
$\quad x_1 = x_0$
$\quad y_1 = x_0$
$\{ y_1 - x_1 \leq 0 \}$
$\quad x_1 \leq 0$
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$\quad x_2 = x_1$
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$\{ \bot \}$

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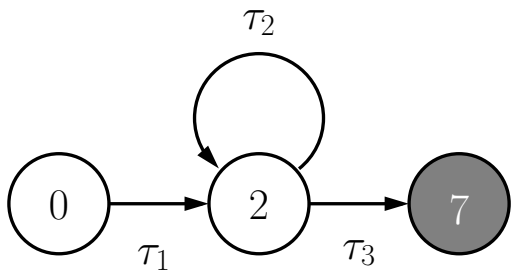

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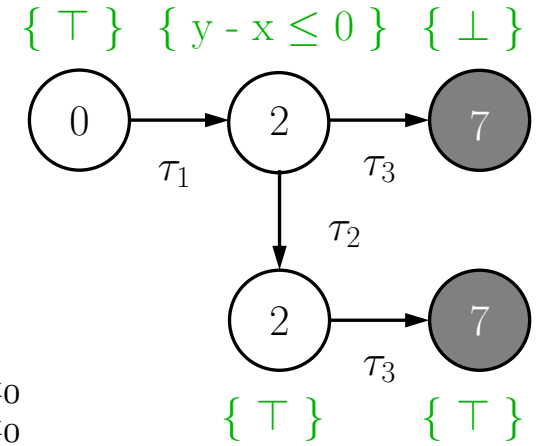

true
$x_1 = x_0$
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$x_1 \geq 1$
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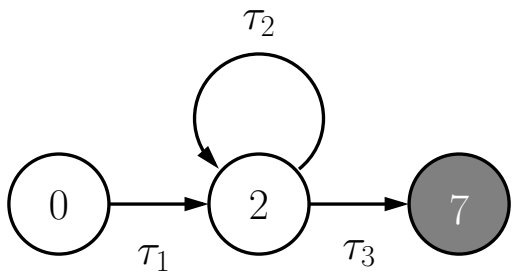

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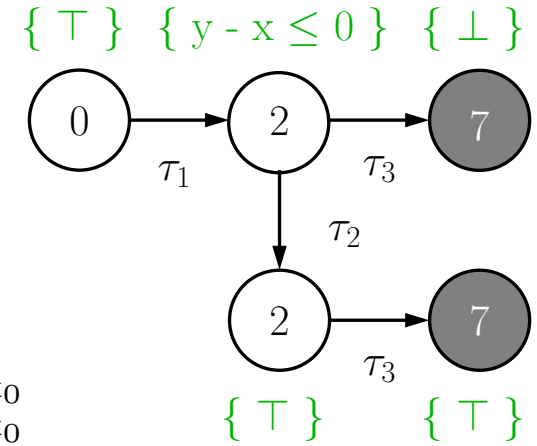

{ ⊤ }
true
$x_1 = x_0$
$y_1 = x_0$

$x_1 \geq 1$
$x_2 = x_1 - 1$
$y_2 = y_1 - 1$

$x_2 \leq 0$
$y_2 \geq 1$
$x_3 = x_2$
$y_3 = y_2$

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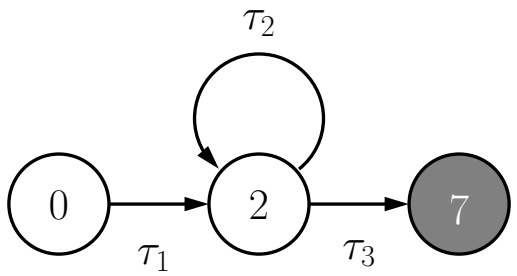

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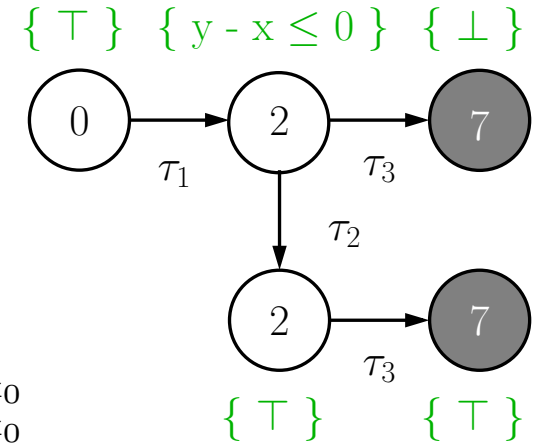

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true
$x_1 = x_0$
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$x_2 = x_1 - 1$
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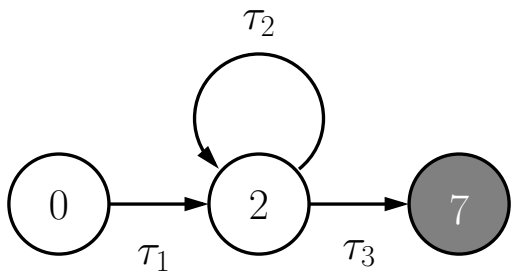

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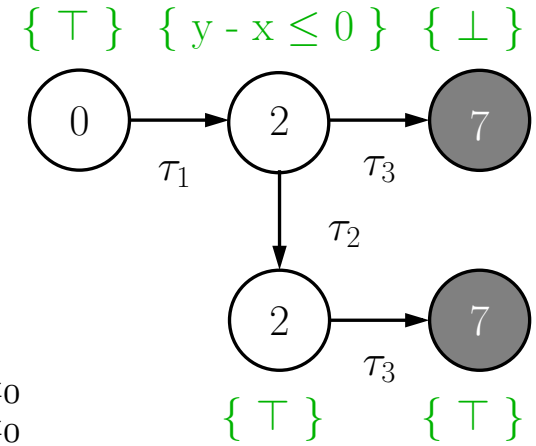

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{ $y_1 - x_1 \leq 0$ }
$x_1 \geq 1$
$x_2 = x_1 - 1$
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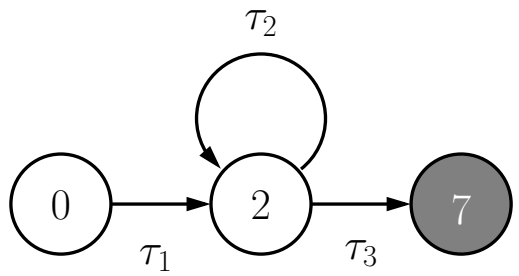

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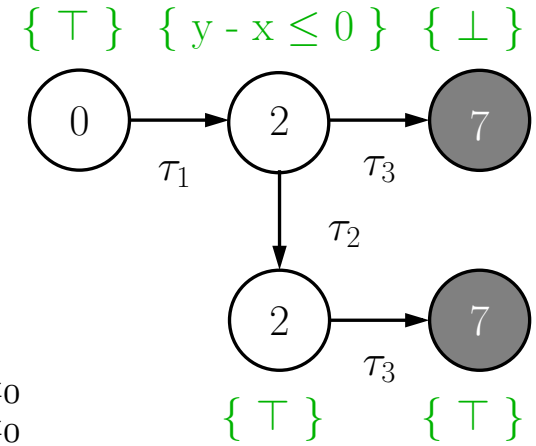

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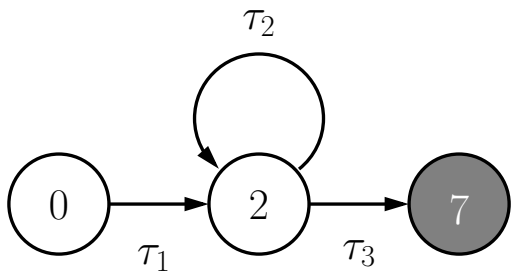

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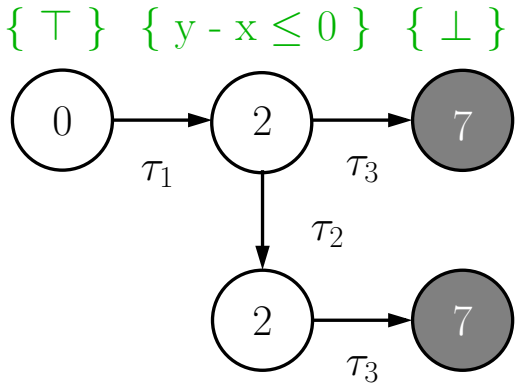

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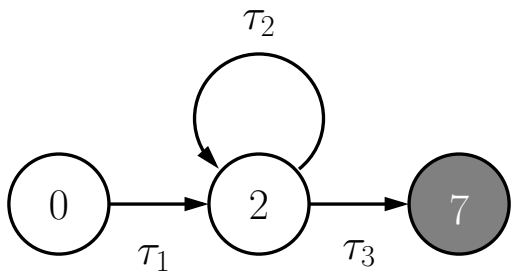

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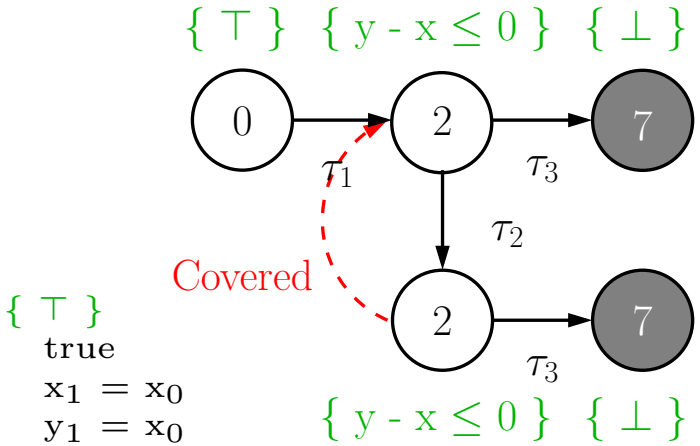

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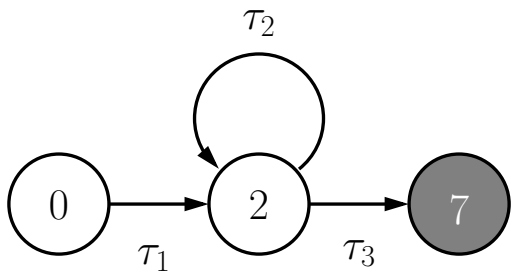

$$\tau_1\colon \top \wedge \left\{ \begin{array}{l} x' := x \\ y' := x \end{array} \right.$$

$$\tau_2\colon x \geq 1 \wedge \left\{ \begin{array}{l} x' := x - 1 \\ y' := y - 1 \end{array} \right.$$

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Lazy Abstraction with Interpolants for Arrays

[ABG+12a]

Extends the Lazy Abstraction with Interpolants framework [McM06]

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Francesco Alberti, Roberto Bruttomesso, Silvio Ghilardi, Silvio Ranise, and Natasha Sharygina.

Lazy abstraction with interpolants for arrays.

In Nikolaj Bjørner and Andrei Voronkov, editors, *LPAR*, volume 7180 of *Lecture Notes in Computer Science*, pages 46–61. Springer, 2012.

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2. Interpolant Manipulation
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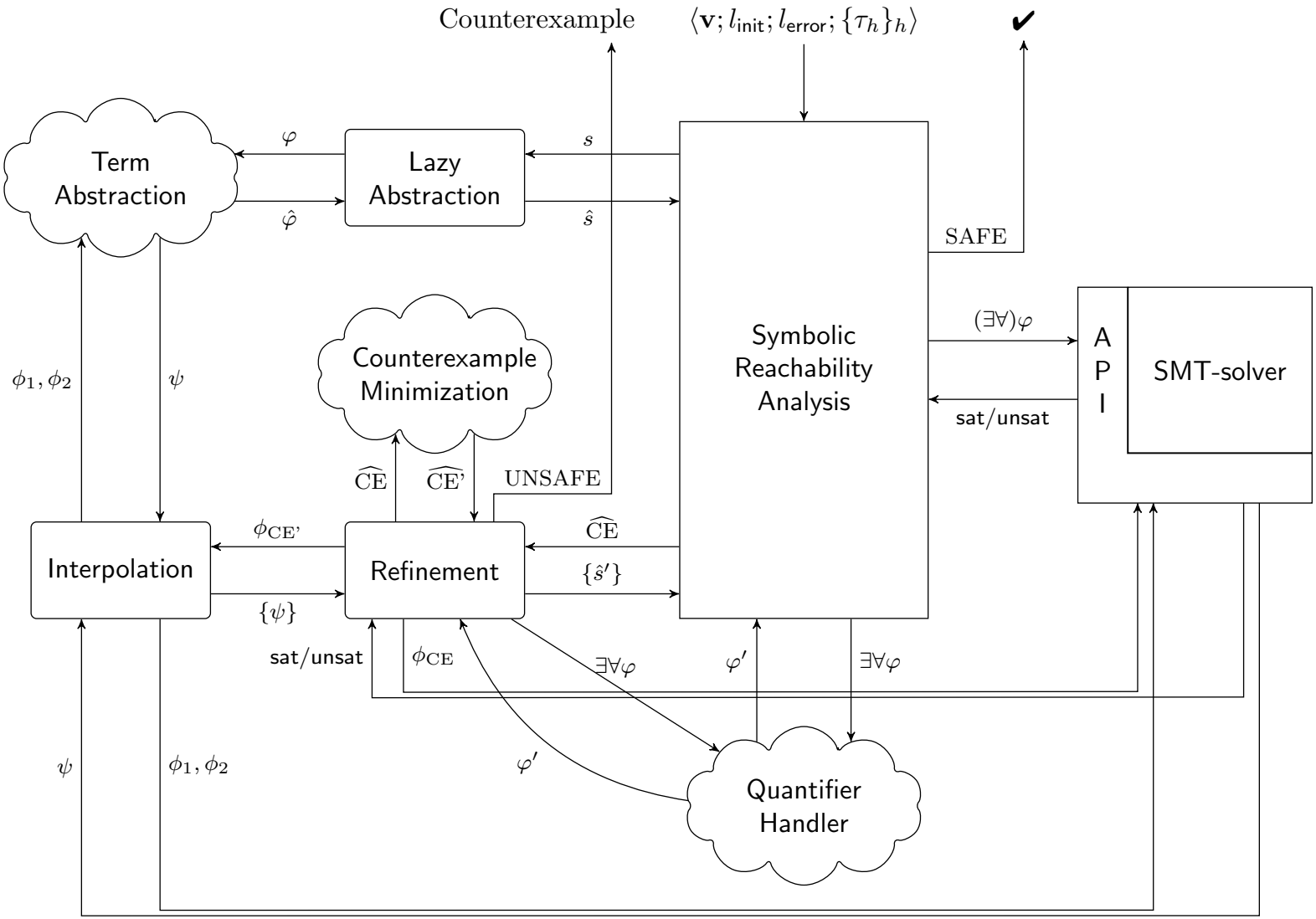
   - Need to select "how" we want to refine

SAFARI architecture



Counterexample Manipulation

Counterexample minimization (CM)

$$
\begin{aligned}
\tau_0 \quad & i_4 = 0 \wedge n > 0 \wedge pc_5 = 1 \wedge rv_4 = rv_5 \\
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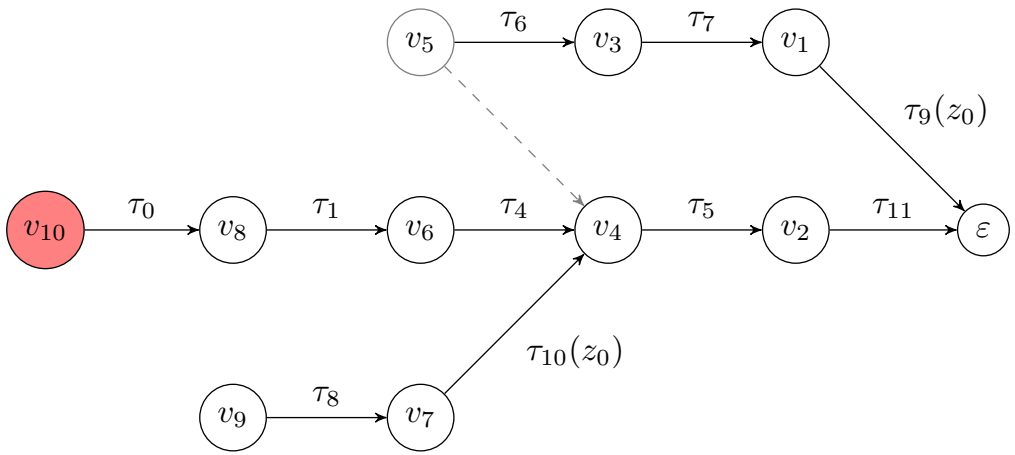

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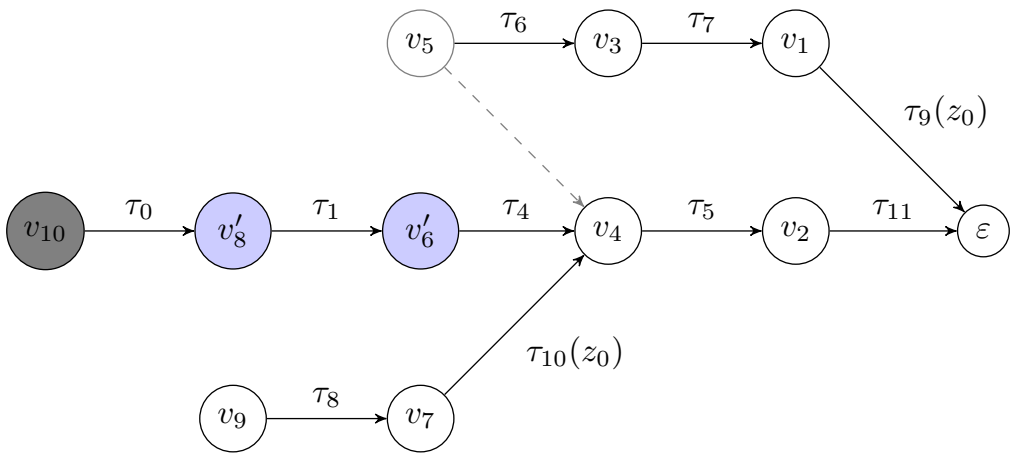

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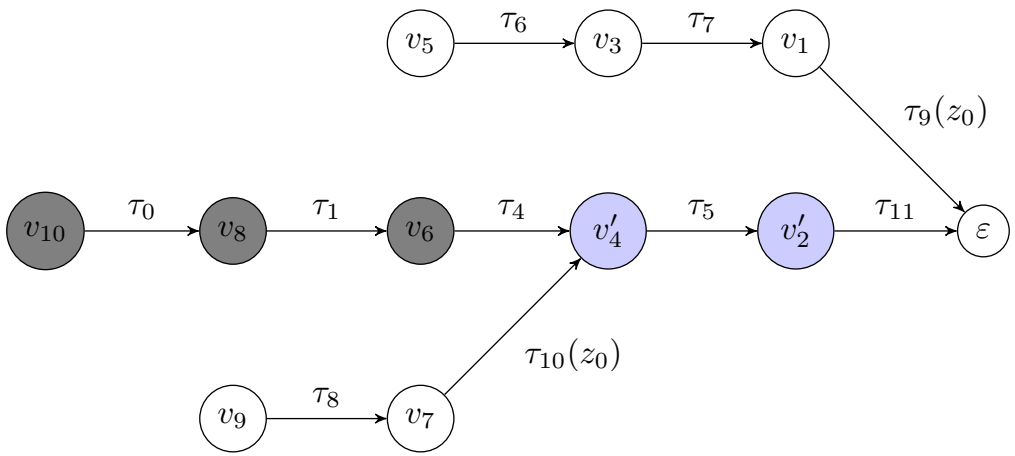

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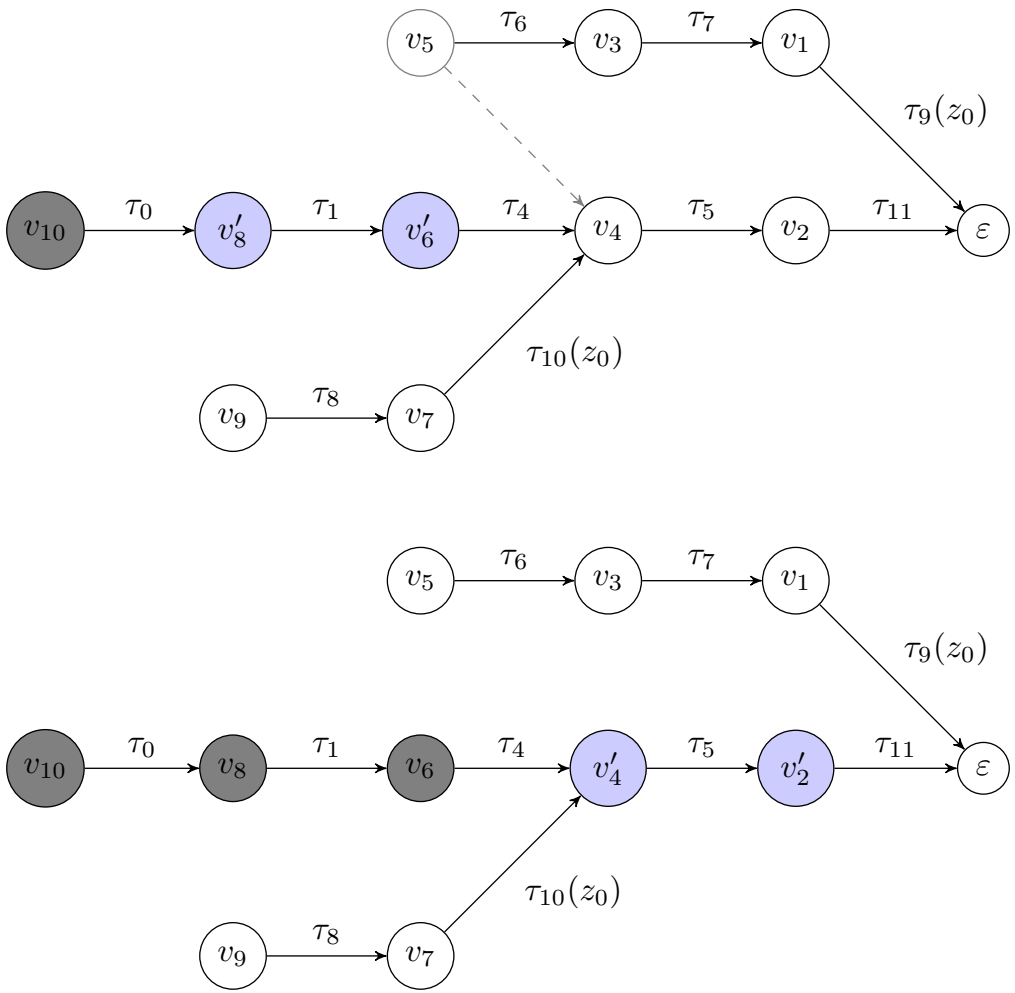

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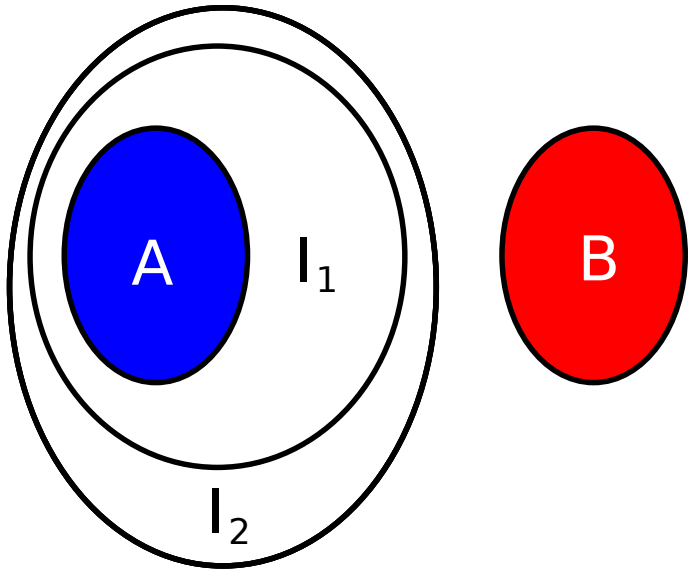

Labeled Interpolation Systems

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- Interpolation parametric in labeling function [DKPW10]

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Labeling Lattice

Labeled Interpolation Systems

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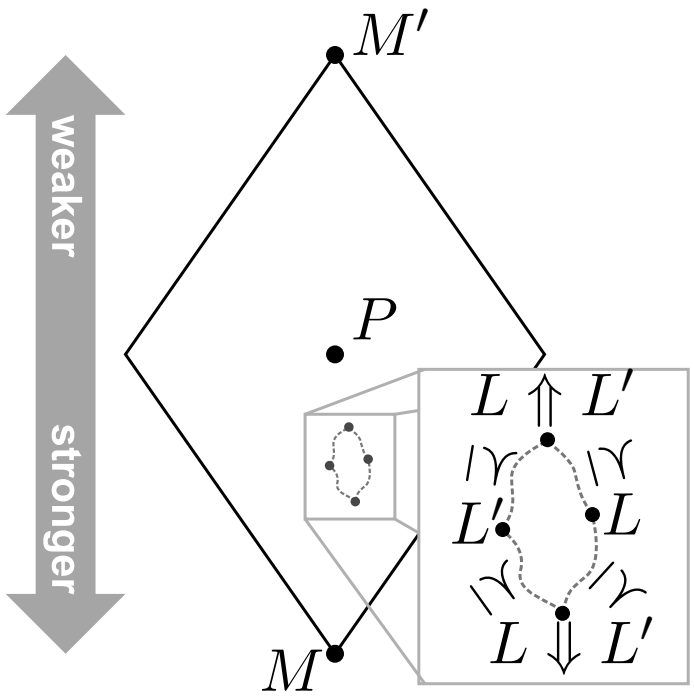

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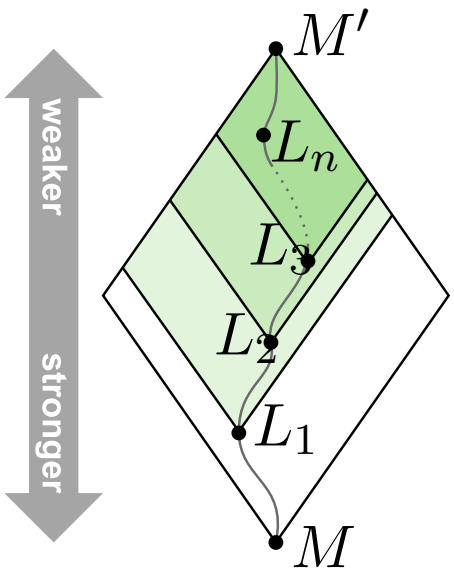

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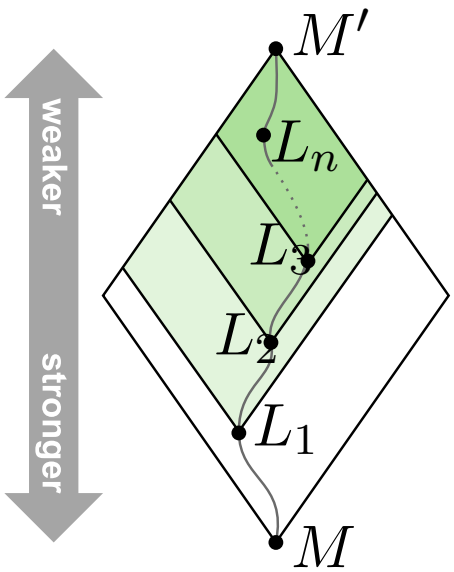
- Always satisfied for $L_1 \equiv \ldots \equiv L_n$ [GRS13]



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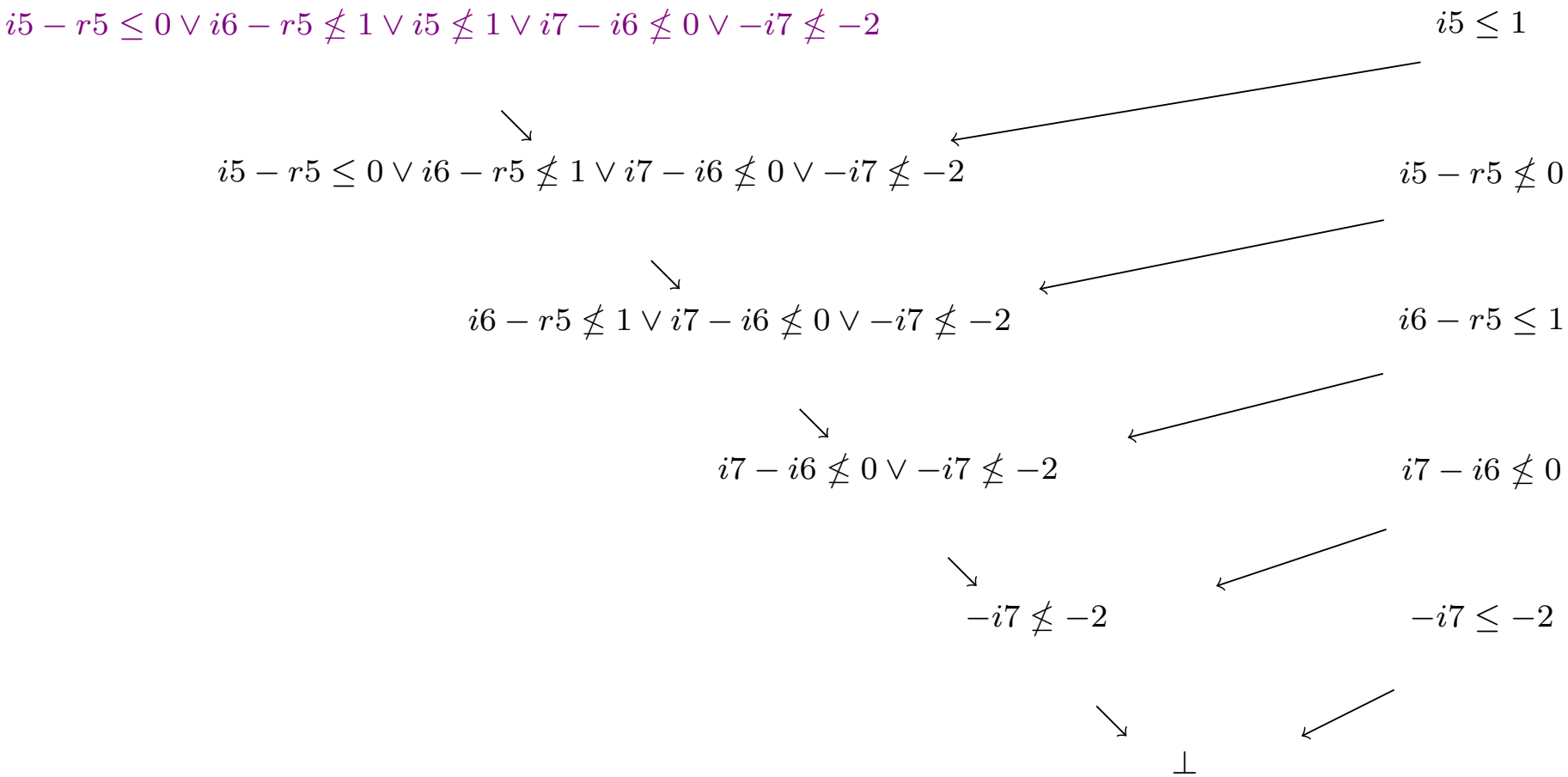
Interpolation in SMT

Example



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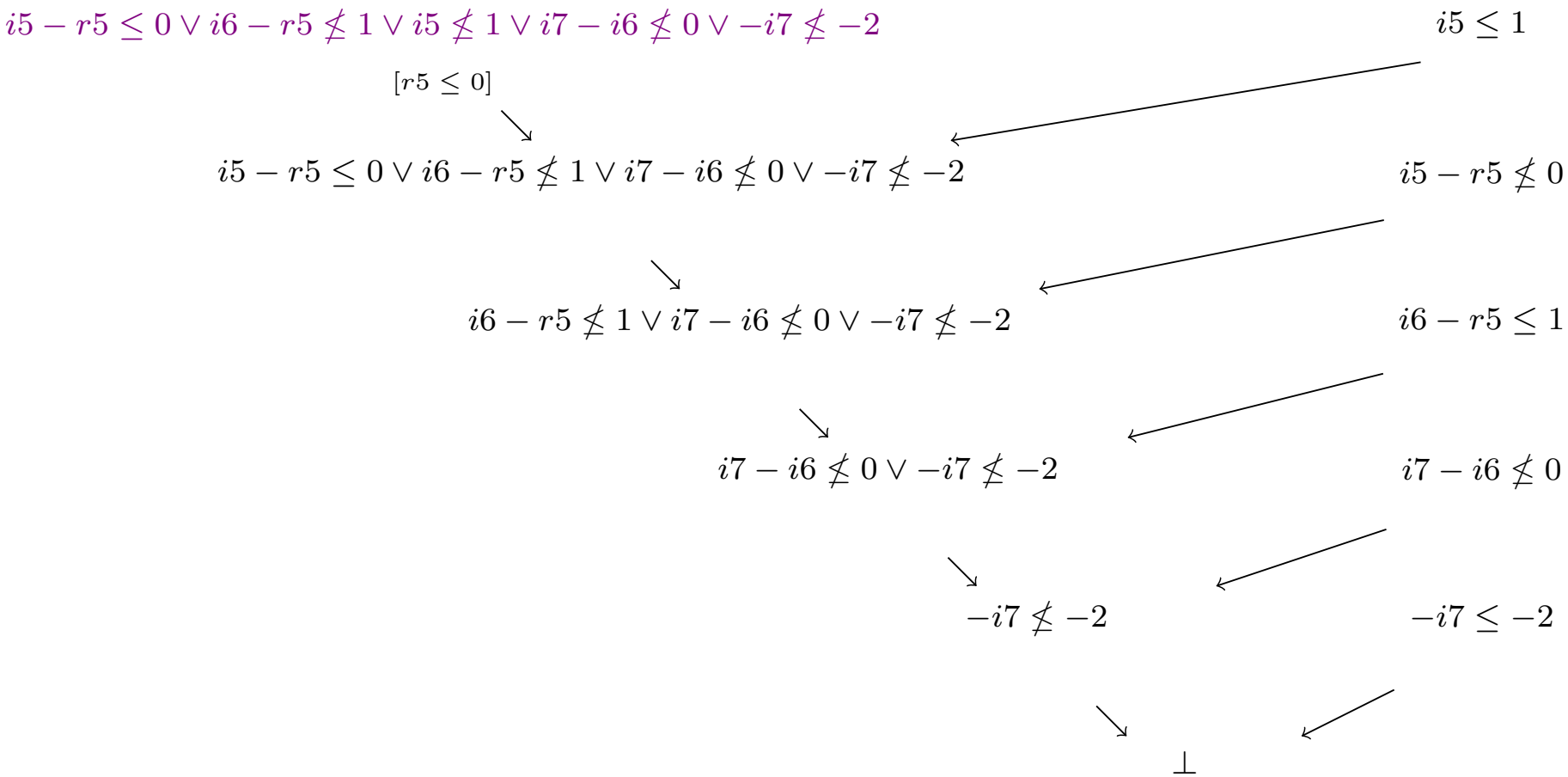

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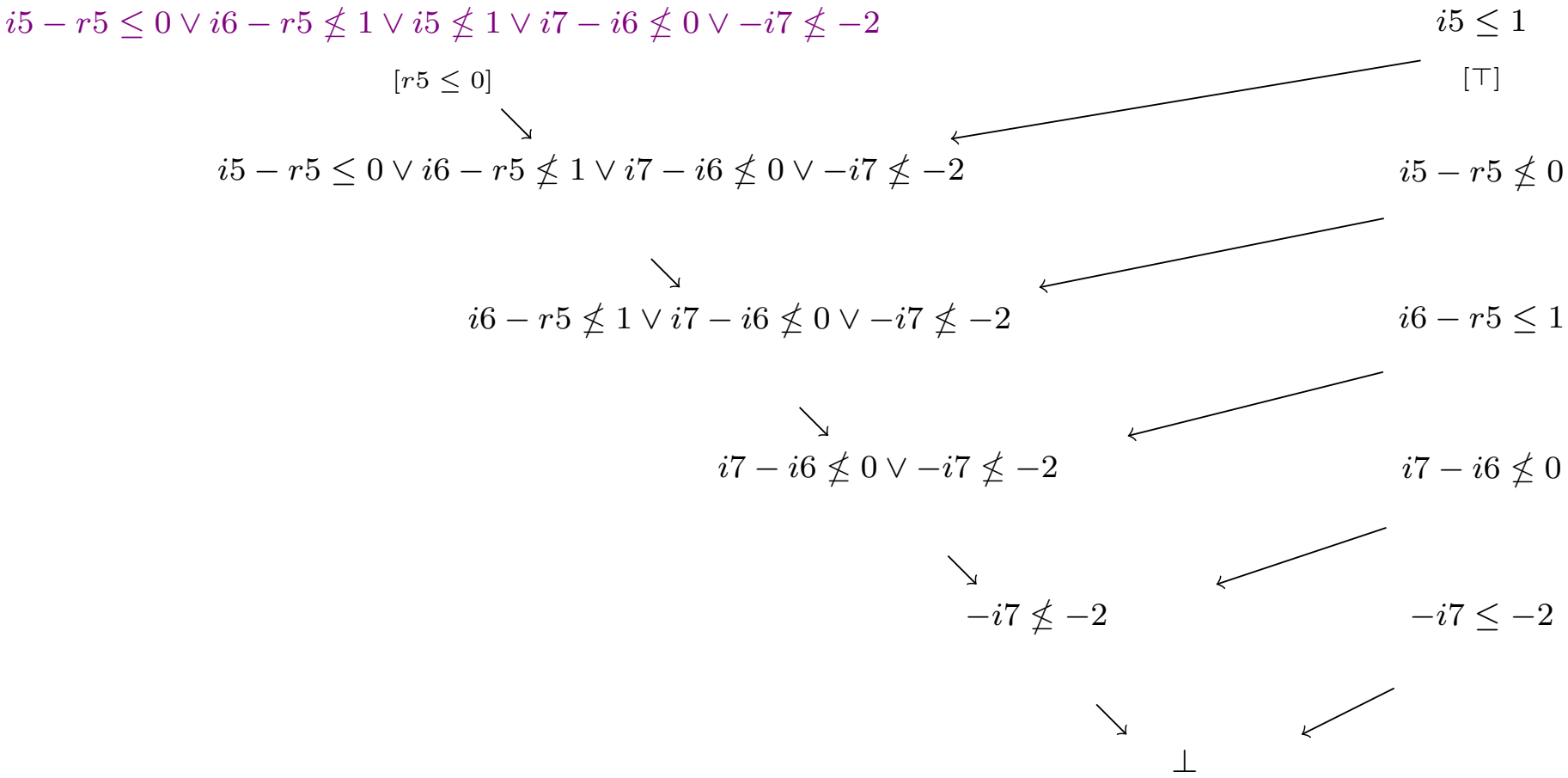

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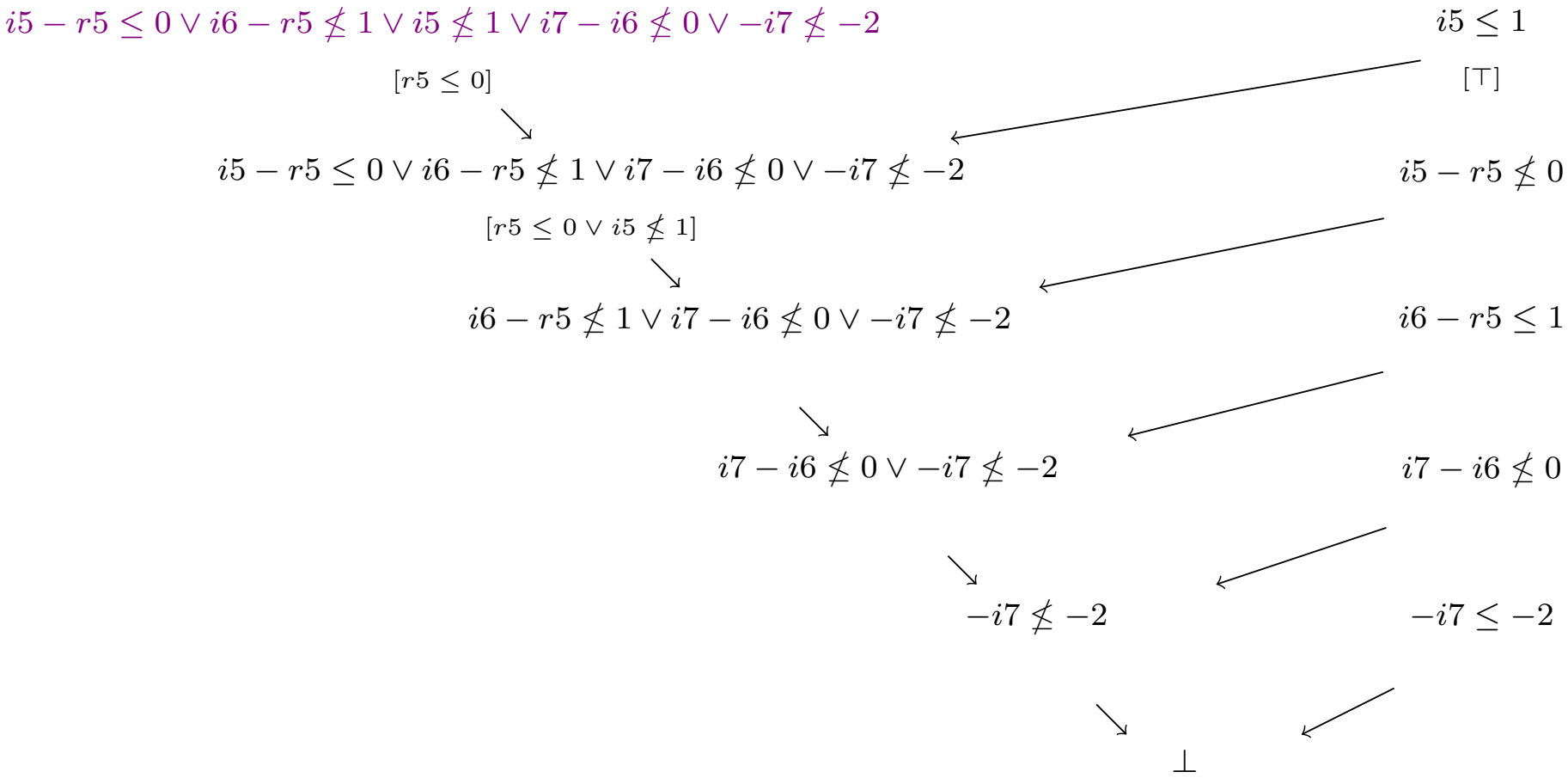

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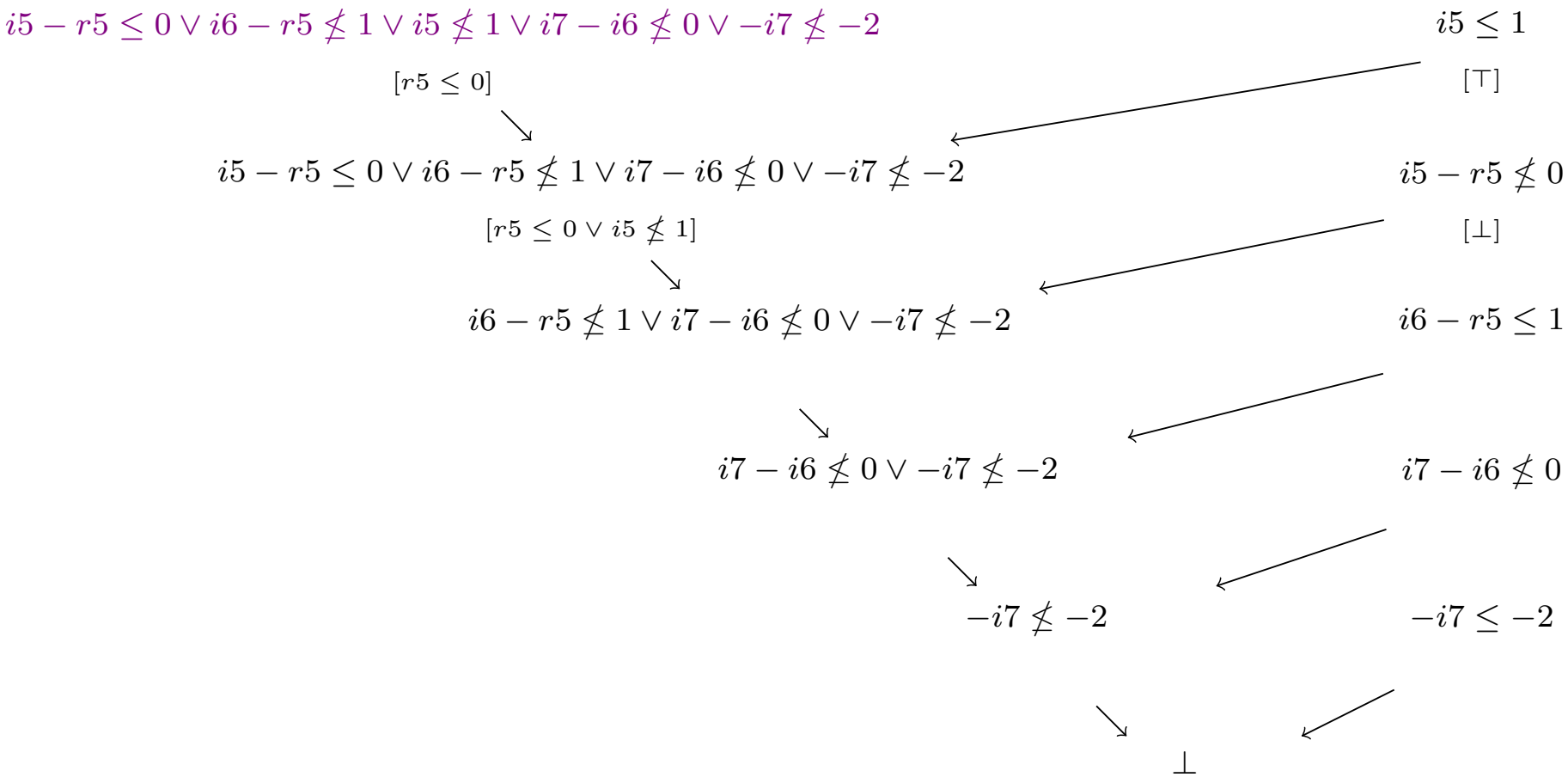

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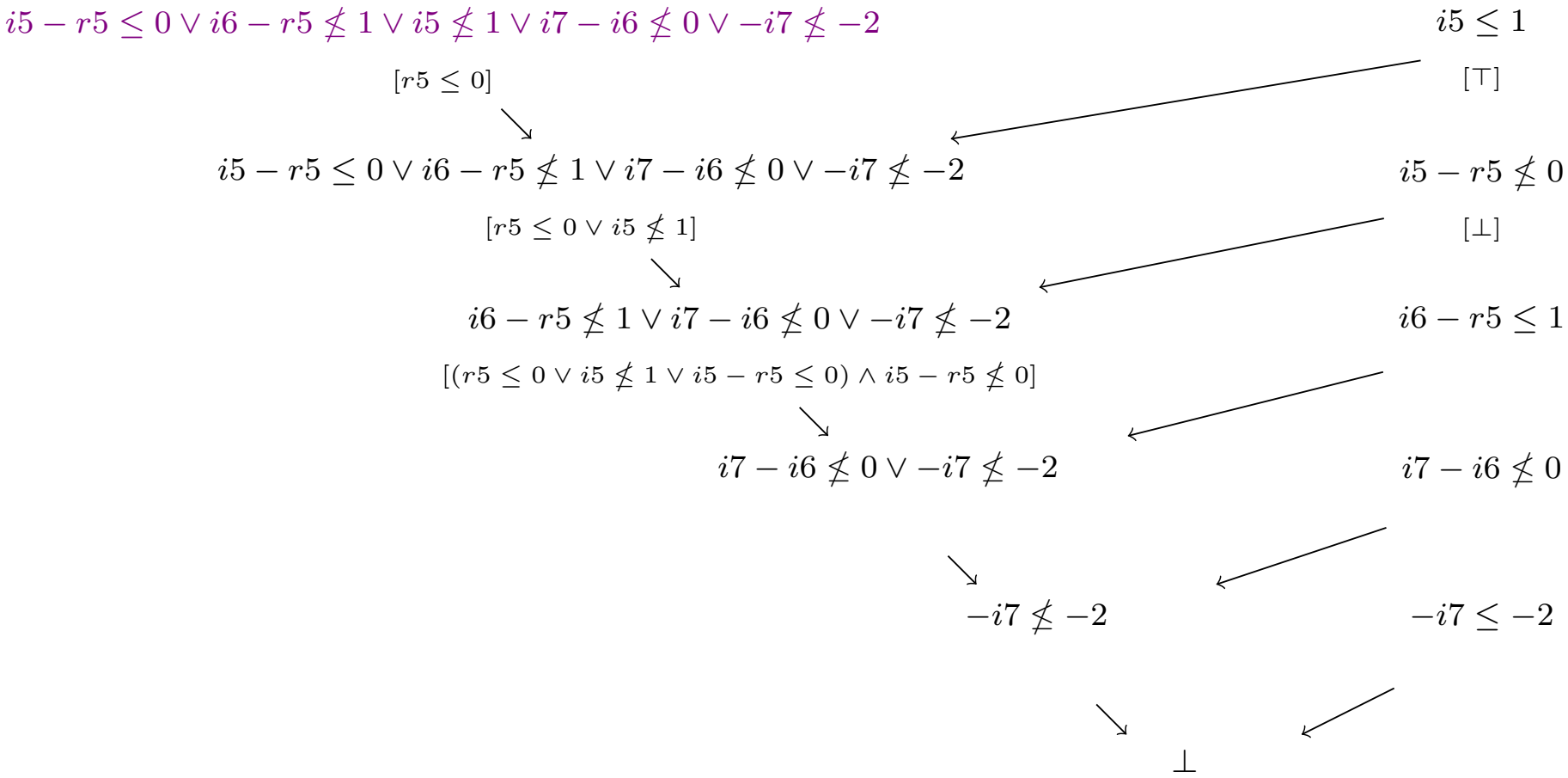

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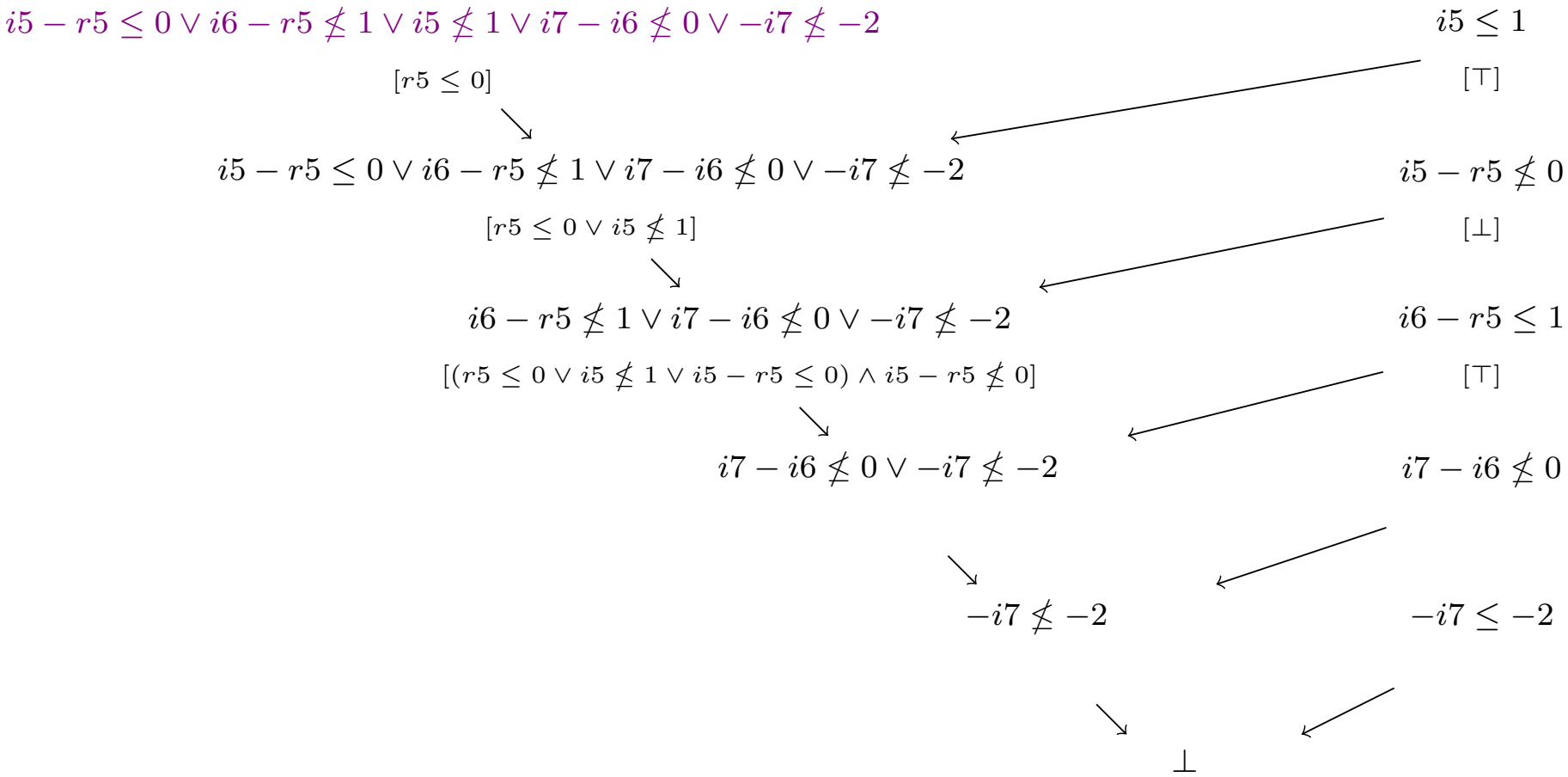

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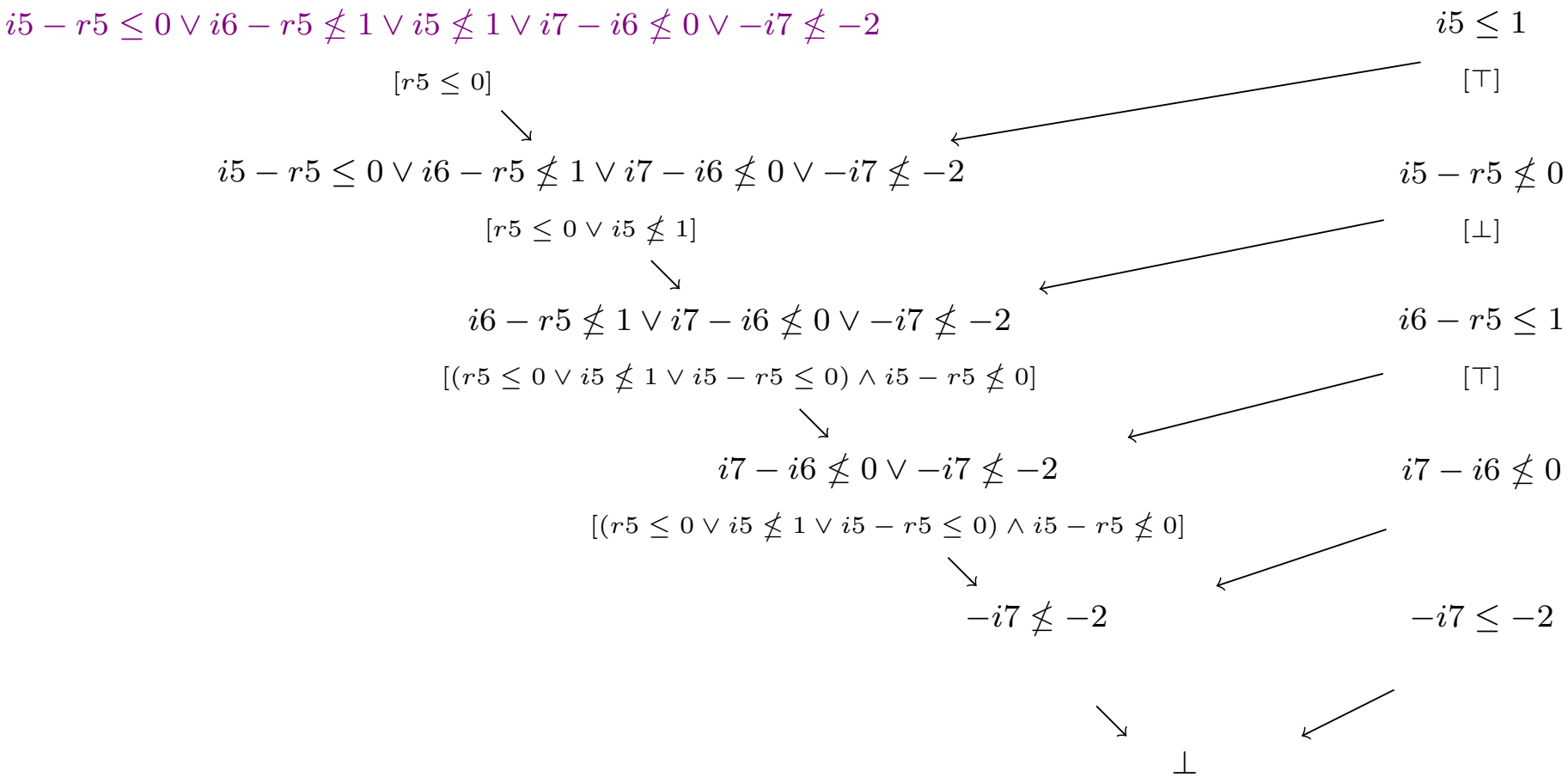

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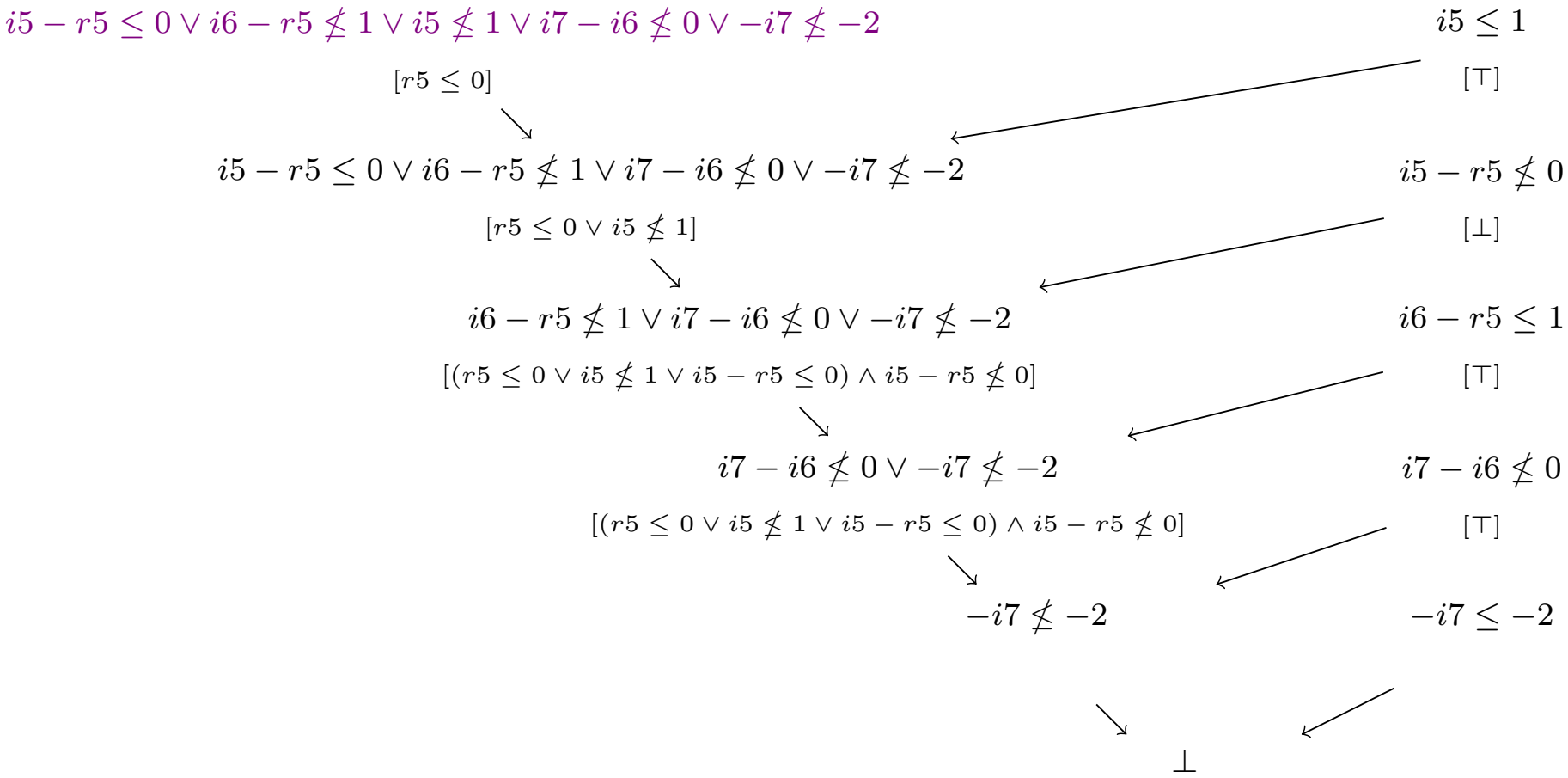

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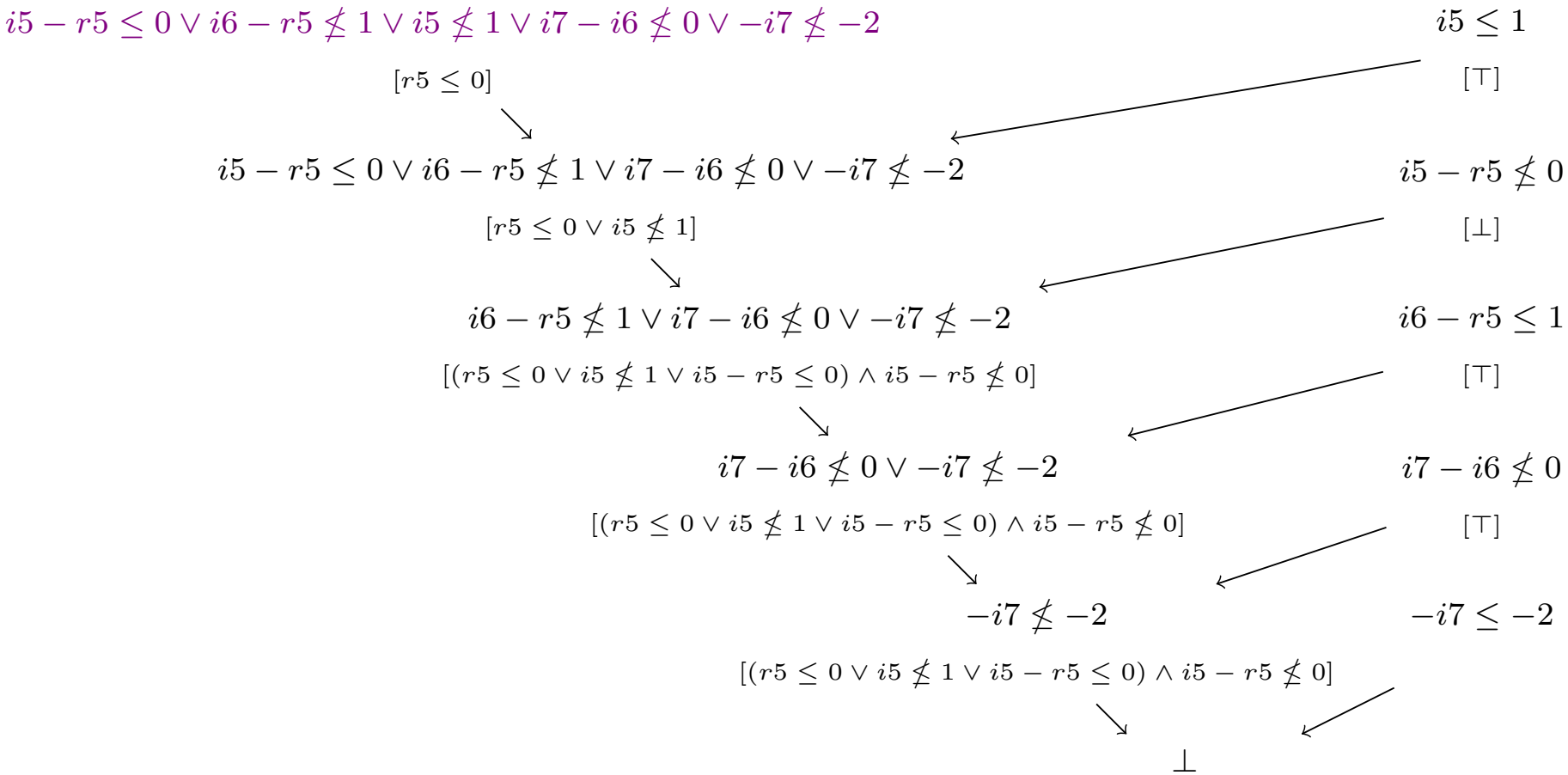

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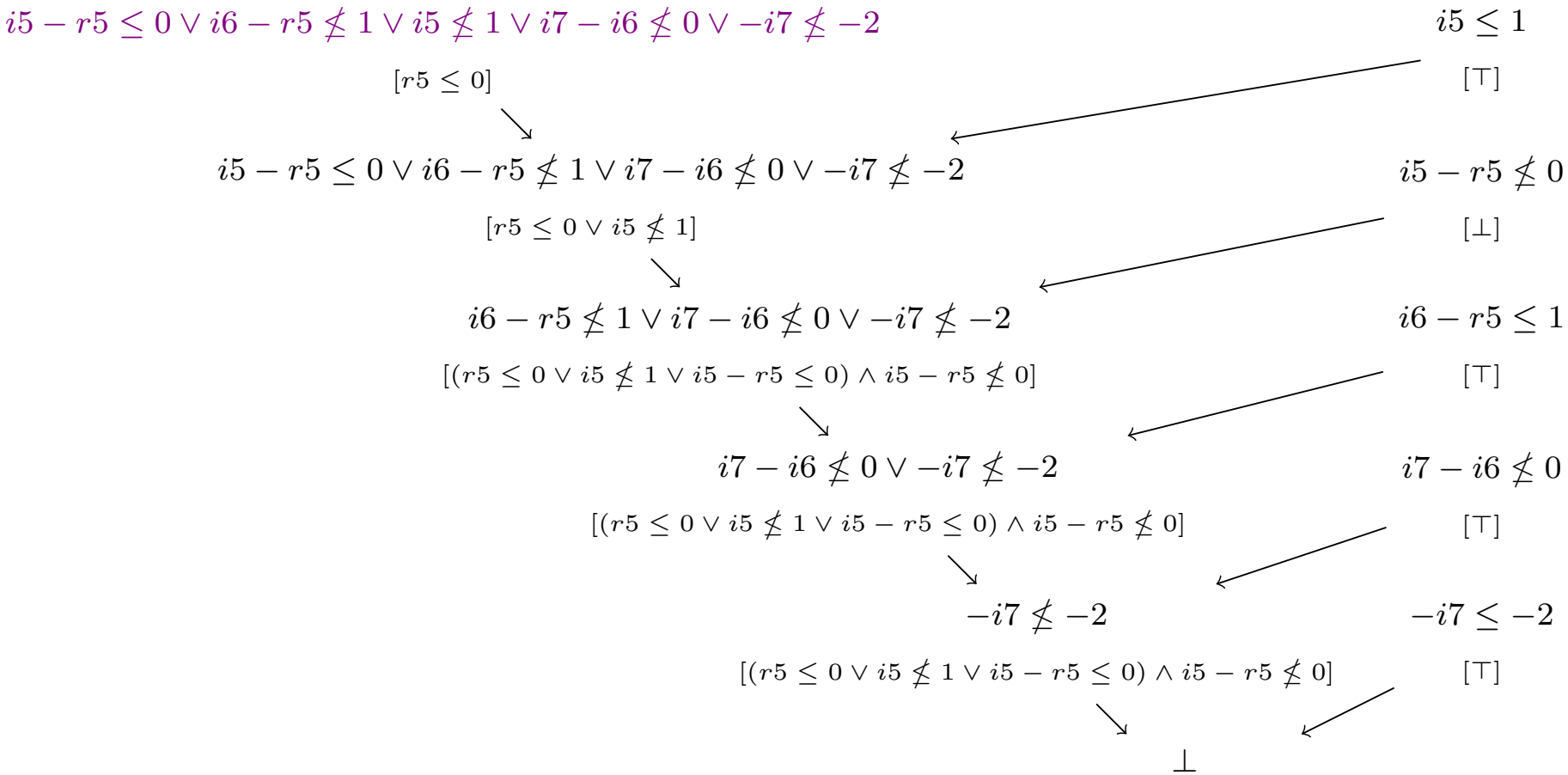

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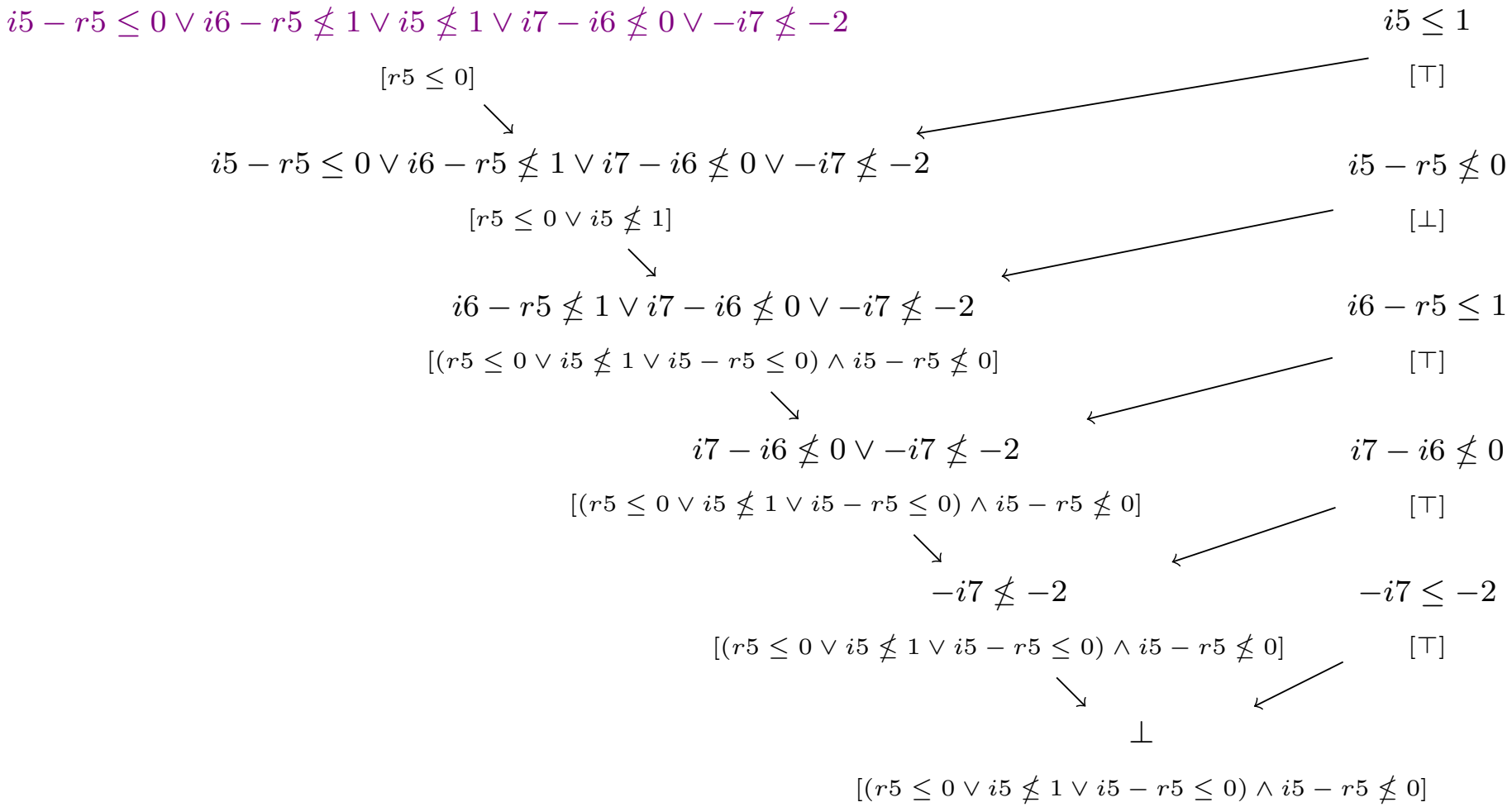

$\mathcal{I}$nteger $\mathcal{D}$ifference $\mathcal{L}$ogic:

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Interpolation in SMT

Theory interpolants in $\mathcal{IDL}$

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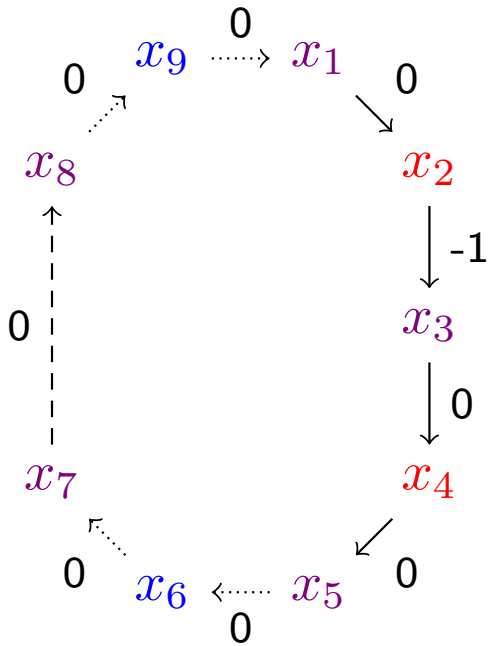

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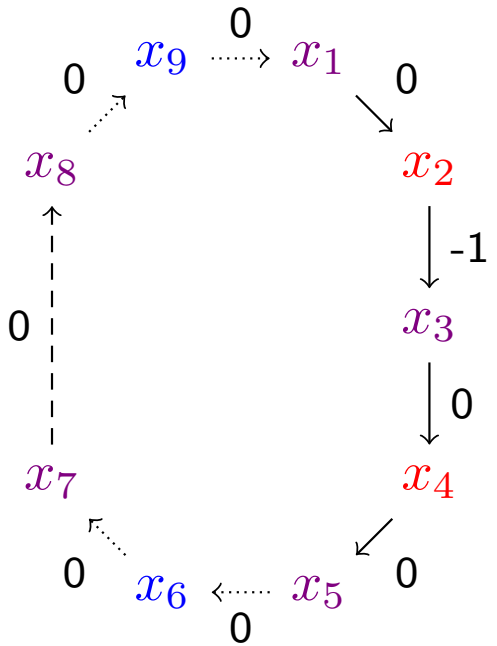

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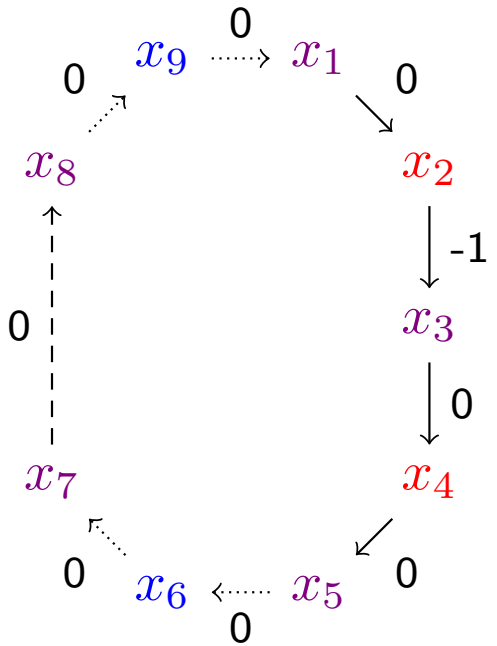
- A, B, AB



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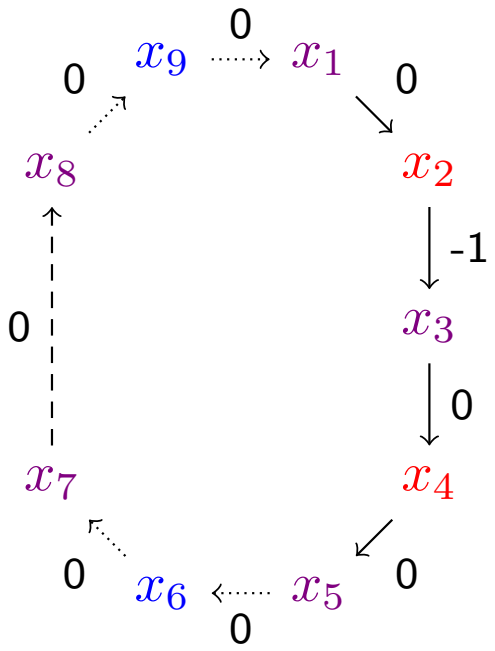

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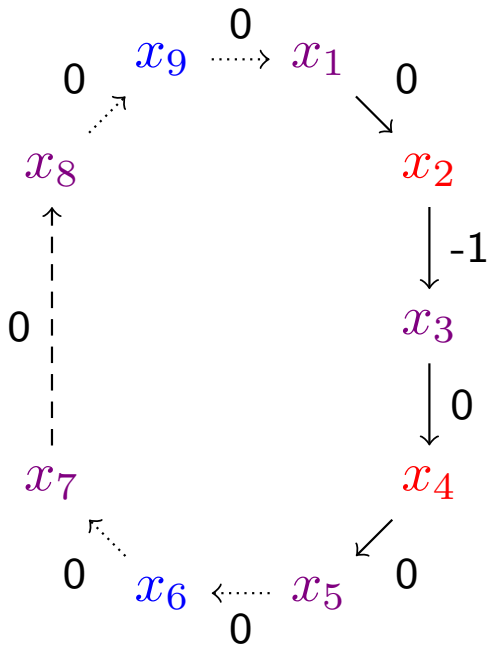

- Interpolant $\equiv$ "summary" of A-edges [CGS10]

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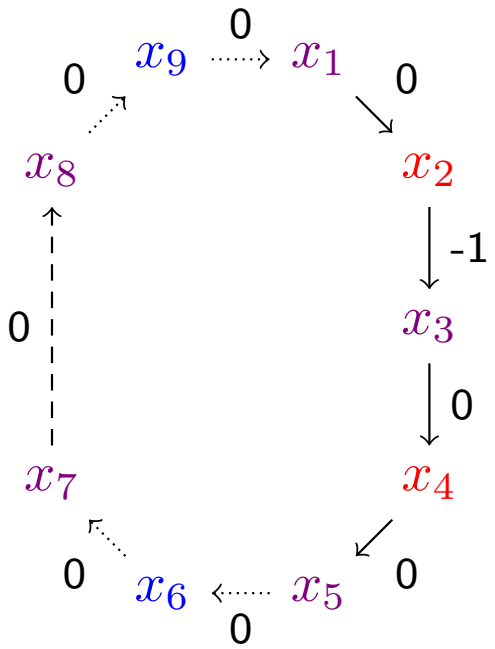

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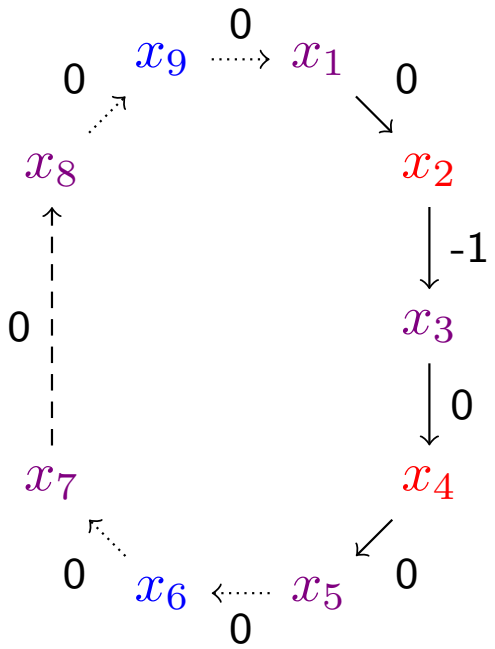
- Degree of summarization, handling of AB-edges

Interpolation in SMT

Theory interpolants in $\mathcal{IDL}$



- Interpolant $\equiv$ "summary" of A-edges [CGS10]
- Degree of summarization, handling of AB-edges
- Full, $x_7 \leq x_8$ to B: $x_1 \leq x_5 - 1$

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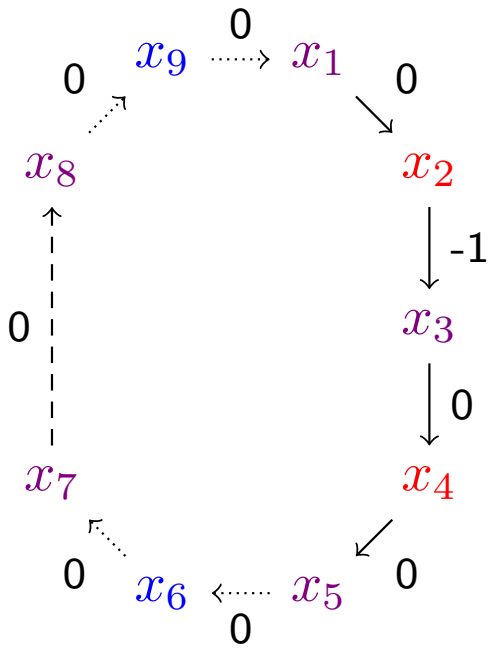

- Interpolant $\equiv$ "summary" of A-edges [CGS10]
- Degree of summarization, handling of AB-edges
- Full, $x_7 \leq x_8$ to B: $x_1 \leq x_5 - 1$
- Partial, $x_7 \leq x_8$ to A: $x_1 \leq x_3 - 1 \wedge x_3 \leq x_5 \wedge x_7 \leq x_8$

Path Interpolation

$\mathcal{IDL}$ Constraints

- Requirement $\tau_1 \rightarrow I_1$ $I_i \wedge \tau_{i+1} \rightarrow I_{i+1}$ $I_{n-1} \wedge \tau_n \rightarrow \bot$

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- Always satisfied for $L_1 \equiv \ldots \equiv L_n$

Theory interpolant manipulation

- Syntactic manipulation of $\mathcal{IDL}$ theory interpolants

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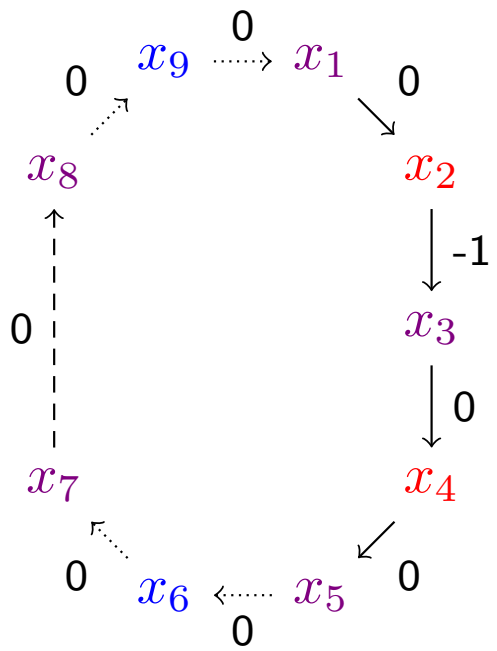

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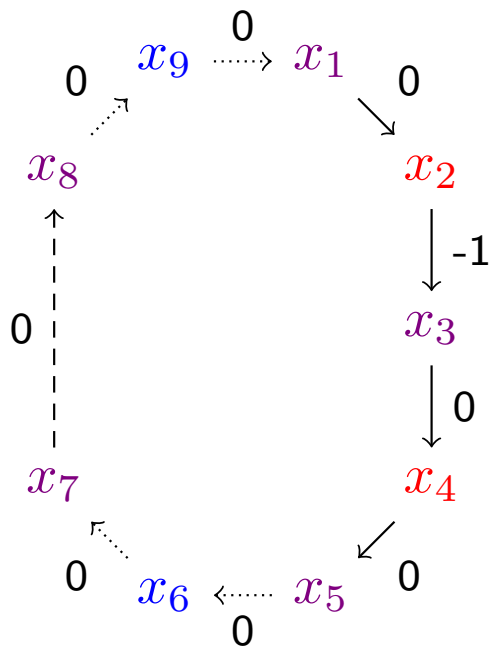

- $x_1 \leq x_3 - 1 \wedge x_3 \leq x_5 \wedge x_7 \leq x_8, \qquad x_1 \leq x_5 - 1$

Path Interpolation in SAFARI

Challenges

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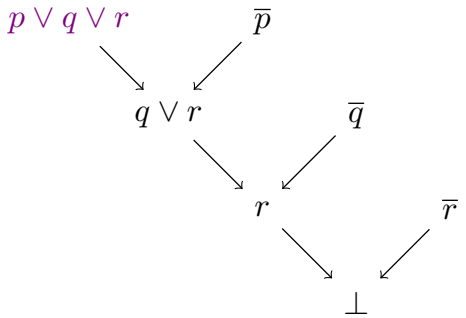

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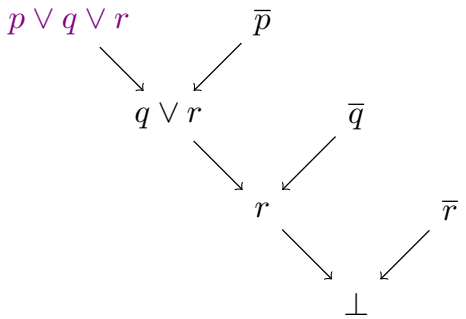

- Very limited impact of interpolant manipulation

Term Abstraction:

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- return $\hat{\phi}_1, \hat{\phi}_2$

Heuristics

CM + TA

Benchmark	Result	No Opt.	TA + CM		Benchmark	Result	No Opt.	TA + CM
binarySort	safe	100	21		p08	safe	100	3
bubbleSort	safe	100	5		p08Test	safe	100	2
compare	safe	100	2		p09	safe	100	3
copy	safe	100	2		p09Test	safe	100	2
copyTest	safe	100	21		p11	unsafe	0	0
filter	safe	100	0		p11Test	unsafe	0	0
filterTest	safe	0	0		p13	unsafe	0	0
find	safe	100	4		p15	unsafe	4	8
findTest	safe	100	29		p15Test	unsafe	3	6
heapAsArray	safe	31	3		p18	unsafe	0	0
init	safe	100	2		p19	unsafe	0	0
initTest	safe	100	11		p19Test	unsafe	0	0
insertionSort	safe	8	5		p20	safe	100	3
maxInArray	safe	100	11		p20Test	safe	100	2
maxInArrayTest	safe	100	7		p21	safe	3	2
p01	safe	100	3		p22	safe	3	2
p01Test	safe	100	2		partInit	safe	100	5
p03	safe	100	3		partition	safe	80	6
p03Test	safe	100	2		partitionTest	safe	100	22
p04	unsafe	0	0		running	safe	100	3
p04Test	unsafe	0	0		vararg	safe	100	2
p06	unsafe	100	2					

Heuristics

CM + TA

Benchmark	Result	Time (s)		Benchmark	Result	Time (s)
binarySort	safe	10.92		p08	safe	0.62
bubbleSort	safe	3.65		p08Test	safe	0.41
compare	safe	0.55		p09	safe	0.75
copy	safe	0.42		p09Test	safe	0.49
copyTest	safe	9.17		p11	unsafe	0.31
filter	safe	0.82		p11Test	unsafe	0.26
filterTest	safe	0.73		p13	unsafe	0.52
find	safe	0.78		p15	unsafe	3.43
findTest	safe	9.93		p15Test	unsafe	2.59
heapAsArray	safe	0.74		p18	unsafe	0.17
init	safe	0.45		p19	unsafe	0.17
initTest	safe	4.22		p19Test	unsafe	0.19
insertionSort	safe	2.07		p20	safe	0.72
maxInArray	safe	4.70		p20Test	safe	0.41
maxInArrayTest	safe	2.34		p21	safe	0.48
p01	safe	0.70		p22	safe	0.51
p01Test	safe	0.45		partInit	safe	1.69
p03	safe	0.72		partition	safe	1.79
p03Test	safe	0.47		partitionTest	safe	18.03
p04	unsafe	0.29		running	safe	0.48
p04Test	unsafe	0.23		vararg	safe	0.46
p06	unsafe	0.54				

Conclusions

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  - Which set of interpolants?
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    - Term abstraction tries to "lift" counterexamples in order to generalize them

$\Rightarrow$ Which parameters do we need in order to drive interpolation-based refinement procedures?

Thank you! Questions?

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